

What Is Claimed Is:

- 1 1. A method for bounding the solution set of a system of linear
2 equations $\mathbf{Ax} = \mathbf{b}$, wherein \mathbf{A} is an interval matrix and \mathbf{b} is an interval vector, the
3 method comprising:
4 preconditioning the set of linear equations $\mathbf{Ax} = \mathbf{b}$ by multiplying through
5 by a matrix \mathbf{B} to produce a preconditioned set of linear equations $\mathbf{M}_0\mathbf{x} = \mathbf{r}$,
6 wherein $\mathbf{M}_0 = \mathbf{BA}$ and $\mathbf{r} = \mathbf{Bb}$;
7 widening the matrix \mathbf{M}_0 to produce a widened matrix \mathbf{M} , wherein the
8 midpoints of the elements of \mathbf{M} form the identity matrix; and
9 using \mathbf{M} and \mathbf{r} to compute the hull \mathbf{h} of the system $\mathbf{Mx} = \mathbf{r}$, which bounds
10 the solution set of the system $\mathbf{M}_0\mathbf{x} = \mathbf{r}$.
- 1 2. The method of claim 1, wherein the method further comprises
2 computing the matrix \mathbf{B} by:
3 computing an approximate center \mathbf{A}_C of the matrix \mathbf{A} ; and
4 forming \mathbf{B} by computing an approximate inverse of \mathbf{A}_C , $\mathbf{B} = (\mathbf{A}_C)^{-1}$.
- 1 3. The method of claim 1, wherein using \mathbf{M} and \mathbf{r} to compute the hull
2 \mathbf{h} involves:
3 forming \mathbf{P} as an inverse of the left endpoint of \mathbf{M} ;
4 forming $c_i = 1/(2P_{ii} - 1)$ for $i = 1, \dots, n$;
5 forming $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$, wherein e_i^T is a unit vector in
6 which the i -th element is 1 and other elements are 0;
7 setting $\inf(h_i) = c_i z_i$ if $z_i > 0$;
8 setting $\inf(h_i) = z_i$ if $z_i \leq 0$; and
9 setting $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$.

1 4. The method of claim 1, further comprising assuring that $\sup(r_i) \geq 0$
2 by changing the sign of r_i (and x_i) if necessary.

1 5. The method of claim 1, further comprising:
2 determining if \mathbf{M} is regular; and
3 using the Gauss-Seidel process for computing the hull \mathbf{h} if \mathbf{M} is not
4 regular.

1 6. A computer-readable storage medium storing instructions that
2 when executed by a computer cause the computer to perform a method for
3 bounding the solution set of a system of linear equations $\mathbf{Ax} = \mathbf{b}$, wherein \mathbf{A} is an
4 interval matrix and \mathbf{b} is an interval vector, the method comprising:
5 preconditioning the set of linear equations $\mathbf{Ax} = \mathbf{b}$ by multiplying through
6 by a matrix \mathbf{B} to produce a preconditioned set of linear equations $\mathbf{M}_0\mathbf{x} = \mathbf{r}$,
7 wherein $\mathbf{M}_0 = \mathbf{BA}$ and $\mathbf{r} = \mathbf{Bb}$;
8 widening the matrix \mathbf{M}_0 to produce a widened matrix \mathbf{M} , wherein the
9 midpoints of the elements of \mathbf{M} form the identity matrix; and
10 using \mathbf{M} and \mathbf{r} to compute the hull \mathbf{h} of the system $\mathbf{Mx} = \mathbf{r}$, which bounds
11 the solution set of the system $\mathbf{M}_0\mathbf{x} = \mathbf{r}$.

1 7. The computer-readable storage medium of claim 6, wherein the
2 method further comprises computing the matrix \mathbf{B} by:
3 computing an approximate center \mathbf{A}_C of the matrix \mathbf{A} ; and
4 forming \mathbf{B} by computing an approximate inverse of \mathbf{A}_C , $\mathbf{B} = (\mathbf{A}_C)^{-1}$.

1 8. The computer-readable storage medium of claim 6, wherein using
2 **M** and **r** to compute the hull **h** involves:
3 forming **P** as an inverse of the left endpoint of **M**;
4 forming $c_i = 1/(2P_{ii} - 1)$ for $i = 1, \dots, n$;
5 forming $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$, wherein e_i^T is a unit vector in
6 which the i -th element is 1 and other elements are 0;
7 setting $\inf(h_i) = c_i z_i$ if $z_i > 0$;
8 setting $\inf(h_i) = z_i$ if $z_i \leq 0$; and
9 setting $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$.

1 9. The computer-readable storage medium of claim 6, wherein the
2 method further comprises assuring that $\sup(r_i) \geq 0$ by changing the sign of r_i
3 (and x_i) if necessary.

1 10. The computer-readable storage medium of claim 6, wherein the
2 method further comprises:
3 determining if **M** is regular; and
4 using the Gauss-Seidel process for computing the hull **h** if **M** is not
5 regular.

1 11. An apparatus that bounds the solution set of a system of linear
2 equations $\mathbf{Ax} = \mathbf{b}$, wherein **A** is an interval matrix and **b** is an interval vector,
3 comprising:
4 a preconditioning mechanism that is configured to precondition the set of
5 linear equations $\mathbf{Ax} = \mathbf{b}$ by multiplying through by a matrix **B** to produce a
6 preconditioned set of linear equations $\mathbf{M}_0 \mathbf{x} = \mathbf{r}$, wherein $\mathbf{M}_0 = \mathbf{BA}$ and $\mathbf{r} = \mathbf{Bb}$;

7 a widening mechanism that is configured to widen the matrix \mathbf{M}_0 to
 8 produce a widened matrix \mathbf{M} , wherein the midpoints of the elements of \mathbf{M} form
 9 the identity matrix; and
 10 a hull computing mechanism that is configured to use \mathbf{M} and \mathbf{r} to compute
 11 the hull \mathbf{h} of the system $\mathbf{M}\mathbf{x} = \mathbf{r}$, which bounds the solution set of the system
 12 $\mathbf{M}_0\mathbf{x} = \mathbf{r}$.

1 12. The apparatus of claim 11, wherein the preconditioning mechanism
 2 is configured to:

3 compute an approximate center \mathbf{A}_C of the matrix \mathbf{A} ; and to
 4 form \mathbf{B} by computing an approximate inverse of \mathbf{A}_C , $\mathbf{B} = (\mathbf{A}_C)^{-1}$.

1 13. The apparatus of claim 11, wherein the hull computing mechanism
 2 is configured to:

3 form \mathbf{P} as an inverse of the left endpoint of \mathbf{M} ;
 4 form $c_i = 1/(2P_{ii} - 1)$ for $i = 1, \dots, n$;
 5 form $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$, wherein e_i^T is a unit vector in
 6 which the i -th element is 1 and other elements are 0;
 7 set $\inf(h_i) = c_i z_i$ if $z_i > 0$;
 8 set $\inf(h_i) = z_i$ if $z_i \leq 0$; and to
 9 set $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$.

1 14. The apparatus of claim 11, wherein the preconditioning mechanism
 2 is configured to assure that $\sup(r_i) \geq 0$ by changing the sign of r_i (and x_i) if
 3 necessary.

1 15. The apparatus of claim 11, wherein the preconditioning mechanism
2 is configured to:
3 determine if \mathbf{M} is regular; and to
4 terminate the process of computing the hull \mathbf{h} if \mathbf{M} is not regular.

1 16. A method for bounding the solution set of a system of linear
2 equations $\mathbf{Ax} = \mathbf{b}$ by multiplying through by the matrix \mathbf{B} to produce a
3 preconditioned set of linear equations $\mathbf{M}_0\mathbf{x} = \mathbf{r}$, wherein $\mathbf{M}_0 = \mathbf{BA}$ and $\mathbf{r} = \mathbf{Bb}$, the
4 method comprising:
5 assuring that $\sup(r_i) \geq 0$ by changing the sign of r_i (and x_i) if necessary;
6 widening the matrix \mathbf{M}_0 to produce a widened matrix \mathbf{M} , wherein the
7 midpoints of the elements of \mathbf{M} form the identity matrix; and
8 using \mathbf{M} and \mathbf{r} to compute the hull \mathbf{h} of the system $\mathbf{Mx} = \mathbf{r}$, which bounds
9 the solution set of the system $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ by,
10 forming \mathbf{P} as an inverse of the left endpoint of \mathbf{M} ,
11 forming $c_i = 1/(2P_{ii} - 1)$ for $i = 1, \dots, n$,
12 forming $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$, wherein e_i^T is a
13 unit vector in which the i -th element is 1 and other elements are 0,
14 setting $\inf(h_i) = c_i z_i$ if $z_i > 0$,
15 setting $\inf(h_i) = z_i$ if $z_i \leq 0$, and
16 setting $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$.

1 17. The method of claim 16, further comprising:
2 determining if \mathbf{M} is regular; and
3 using the Gauss-Seidel process for computing the hull \mathbf{h} if \mathbf{M} is not
4 regular.

- 1 18. The method of claim 16, wherein the method further comprises
2 computing the matrix **B** by:
3 computing an approximate center A_C of the matrix **A**; and
4 forming **B** by computing an approximate inverse of A_C , $B = (A_C)^{-1}$.